Design of Tesla’s Two-Phase Inductor

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Abstract—This paper describes a new method for designing Tesla’s two-phase inductor for demonstration of rotating magnetic field, based on the famous Tesla’s Egg of Columbus experiment. The design is based on electromagnetic and thermal analytical models of the two-phase inductor and verified by computer simulation and FEM analysis.

Keywords—FEM analysis; rotating magnetic field; single-phase induction motor; Tesla’s Egg of Columbus;

NOMENCLATURE

\( D, d \) Outer and inner core diameter [mm],
\( a, h \) Iron core width and height [mm],
\( \mu_0 = 4\pi \times 10^{-7} \) Permeability of free space [H/m],
\( \omega, f \) Angular frequency [rad/s] and frequency [Hz],
\( \alpha, \beta \) Phase index,
\( N_{on}, N_{of}, N \) Number of turns per phase,
\( m = N_f/N_a \) Transfer coefficient,
\( N_{1/2} = N_{1/2} \) Number of turns per half-winding,
\( \kappa_d \) Equivalent reluctance of air gap [H^-1],
\( X_w, X_g, X \) Reactance of phase winding [Ω],
\( X_{1/2} = X_{1/2} \) Reactance of one half-winding [Ω],
\( R_w, R_g, R \) Resistance of phase winding [Ω],
\( Z_w, Z_g, Z \) Impedance of phase winding [Ω],
\( X_C, R_C \) Capacitor reactance and resistance [Ω],
\( U_C, C \) Capacitor voltage [V] and capacitance [μF],
\( \rho_{Cu} = 1/57 \) Specific resistance of copper at 20°C [Ωmm²/m],
\( l_{Cu}, d_{Cu}, d_{Cu} \) Average length [mm],
\( n_{Cu}, n_{Cu} \) Copper wire cross section [mm²],
\( n_{Cu}, n_{Cu} \) Copper wire diameter [mm],
\( n_{Cu}, n_{Cu} \) Number of conductor layers,
\( I_{ph}, I \) RMS value of phase and total current [A],
\( U_{ph}, U \) RMS values of phase and supply voltage [V],
\( F_{ph}, F \) Amplitude of magneto-motive force [Atumns],
\( B_{ph}, B_{ph}, B_g \) Amplitude of air gap magnetic flux [Wb],
\( \Delta, \rho_{Cu} \) Amplitude of air gap magnetic flux density [T],
\( m_{Cu}, V_{Cu} \) Current density [A/mm²] and copper losses [W],
\( \gamma_{Cu} = 8900 \) Total mass [kg] and volume of copper [m³],
\( \rho_{Cu} \) Specific mass of copper [kg/m³],
\( \theta, \theta_a \) Operating and ambient temperature [°C],
\( \theta_0 = \theta + \theta_0/2 \) Average (film) temperature [°C],
\( \Delta \theta = \theta - \theta_a \) Temperature increase [°C],
\( \alpha_e, \alpha_t \) Heat transfer coefficients [W/m²°C],
\( S_{th}, S_r \) Heat transfer surface areas [mm²],
\( \sigma = 5.674 \times 10^{-8} \) Stefan-Boltzmann constant [W/m²K⁴],
\( \varepsilon \) Emissivity coefficient,
\( Nu, Gr, Pr \) Nusselt, Grashoff and Prandtl number,
\( s_f \) Thermal conductivity of air [W/m°C],
\( \gamma_f \) Kinematic viscosity of air [m²/s],
\( g = 9.81 \) Acceleration of free fall [m/s²],
\( \beta = (235 + \theta_0)^{1/2} \) Thermal expansion coefficient [ºC-¹],
\( \gamma_f = 1.2928 \) Specific mass of air [kg/m³],
\( c_f = 1005 \) Specific thermal capacity of air [J/kg/ºC].

I. INTRODUCTION

In February 1882, the Serbian inventor and scientist Nikola Tesla came up with the design of the rotating magnetic field and in 1888 he constructed the first induction motor. Tesla’s first induction motor had a toroidal stator with a two-phase inductor, unlike a cylindrical stator with a three phase inductor which is nowadays a standard design. In order to demonstrate the effects of the rotating magnetic field at the World fair exhibition in Chicago in 1893, Tesla used his two-phase inductor to lift and spin a metal egg, thus “performing the feat of Columbus without cracking the egg”[1]. Tesla’s original design of the two-phase inductor is shown in Fig. 1.

Figure 1. The original design of the two-phase Tesla inductor [2]
Two coils are wound on an iron core of toroidal shape. The two phase coils are displaced by 90 (geometrical) degrees and are supplied with two alternating currents phase shifted by 90 (electrical) degrees. In this way a rotating magnetic field is generated in the air gap. If a metal object, e.g. a copper or aluminum egg, is placed on a platform in the centre, it will start to rotate, and eventually stand on its main axis, due to gyroscopic effect. Tesla’s two-phase inductor is in fact the stator of a two-phase induction motor, with the metal egg being the rotor.

Replicas of “Tesla’s Egg of Columbus” can be found in museums dedicated to Tesla and almost every Faculty of electrical engineering has its own unique copy. There are even companies that create authentic replicas for museums and private collectors [3]. These replicas are not difficult to reproduce, however in most cases they deviate from Tesla’s original design. For example in [4] using the laws of electrical machine resemblance, a stator of a conventional four-pole, three-phase induction motor was redesigned to demonstrate the “Egg of Columbus” experiment, and in [5] an electronically controlled rotating magnetic field inductor, using power switches, oscillator and counter circuits is presented. All these replicas don’t properly illustrate the brilliance and simplicity of the rotating magnetic field concept and they are not a practical educational tool which, in a simple way, conveys basic electro-technical engineering principles to young students.

This paper gives the design and construction principles of Tesla’s two phase inductor. The paper is organized as follows. In Section II the analytical model for electromagnetic design is presented and proper level of magnetic flux density in air gap is selected. Equations for calculating the phase reactance and resistance, required capacitance in each phase are derived. Section III gives a method for determining adequate current density, based on analytical thermal analysis. Using the derived equations, all the necessary parameters are calculated in Section IV and the results of the design are verified via computer simulation and Finite Element Method (FEM) analysis. Conclusion is given in Section V.

II. ELECTROMAGNETIC DESIGN

The design of electrical machines is an iterative procedure based on calculations and assumptions on the first step, and on necessary corrections and experimental verification on the second step. Assuming fixed dimensions of used iron core, the electromagnetic design starts with the selection of magnetic flux and current density and ends by calculating number of turns in each phase.

A. Magnetic Circuit

The magnetic circuit for one phase winding of the Tesla inductor is shown in Fig. 2. The phase winding is divided into two halves which are connected in series, but wound in opposite directions and placed on opposing sides of the iron core. The resulting magnetic flux $\Phi_{g}$ passes through the air gap, while only half of the resulting flux $\Phi_{fe} = \Phi_{g}/2$ passes through each side of the iron core. For design analysis, the magnetic circuit in Fig. 2 can be represented with an equivalent circuit, shown in Fig. 3. The magnetic circuit equivalent takes only the equivalent magnetic resistance (reluctance) of the air gap into account, while the reluctance of the iron core is neglected due to high values of iron core permeability.

\[
X_{1/2} = \alpha \frac{N_{1/2}^2}{\frac{3}{\pi} \cdot \mu_{0} \cdot h \cdot \omega \cdot N_{1/2}^2} \quad (1)
\]

where the equivalent air gap reluctance is:

\[
r_{g} = \frac{3}{\pi \cdot \mu_{0} \cdot h} \quad (2)
\]

Since two half-windings are connected in series to form one phase winding, after introducing frequency and number of turns per phase in (1), the following equation for reactance of each phase winding is obtained:

\[
X = 2 \cdot X_{1/2} = \frac{1}{3} \cdot \pi^2 \cdot \mu_{0} \cdot h \cdot f \cdot N^2 \quad (3)
\]

B. Phase Impedance

Assuming a homogeneous magnetic field in the air gap, the phase reactance can be calculated by using similar approach to slot leakage reactance calculation for standard induction motors [6]. Later in the text, by using computer simulation and FEM analysis, it will be shown that this is a very good approximation. The reactance of one half-winding of each phase, is:
The phase resistance at $\theta_0 = 20°C$ can be calculated as:

$$R_0 = \rho_{Cu0} \left( \frac{L_{Cu}}{S_{Cu}} \right) = \rho_{Cu0} \left( \frac{N \cdot I_{Cu}}{S_{Cu}} \right)$$

(4)

The cross section of copper wire in (4) is:

$$S_{Cu} = \frac{\pi}{4} \cdot d_{Cu}^2$$

(5)

The average length of copper wire is:

$$L_{Cu} = 2 \cdot N_{l/2} \cdot I_{Cu} = 2 \cdot N \cdot (a + h \cdot 2 \cdot n \cdot d_{Cu})$$

(6)

The coil resistance at operating temperature $\theta$ is:

$$R = R_0 \cdot \frac{235 + \theta}{235 + \theta_0}$$

(7)

The impedance of each phase winding is then:

$$Z = \sqrt{R^2 + X^2}$$

(8)

In equations (1)-(8) phase indexes have been left out, meaning that the equations are valid for both phases. In order to calculate the parameters for a particular phase, one must use the number of turns, number of conductor layers and cross section of that particular phase winding.

C. Permanently Connected Capacitor

In order to get a symmetrical rotating magnetic field, it is needed to have equal amplitudes of Magneto-Motive Forces (MMFs) for both phases, and an electrical phase delay of 90° between the phase currents. In standard single-phase induction motors, this is achieved by using permanently connected capacitor in one of the phases, e.g. phase $\beta$ in Fig. 4. For calculating the capacitance in phase $\beta$ a standard procedure described in [7] will be used.

The transfer ratio is calculated using [7]:

$$m = \frac{X_a + \sqrt{X_a^2 - 4 \cdot R_C \cdot R_a}}{2 \cdot R_a}$$

(9)

while capacitor reactance and capacitance are [7]:

$$X_C = m^2 \cdot X_a + m \cdot R_a$$

(10)

$$C = \frac{1}{\omega \cdot X_C} = \frac{1}{2 \cdot \pi \cdot f \cdot X_C}$$

(11)

While calculating the transfer ratio, one must assume the value of capacitor resistance, but for rough calculations, the capacitor resistance can be neglected.

D. Number of Turns per Phase

The magnetic flux density in the centre of the air gap due to excitation from one phase can be calculated from Fig. 3. For that purpose, the Kapp-Hopkins law for magnetic circuits is used:

$$\Phi_g = \frac{F}{3R_g}$$

(12)

The Magneto-Motive Force (MMF) generated from both phases is:

$$F = \frac{\sqrt{2}}{2} \cdot N_a \cdot I_a = \frac{\sqrt{2}}{2} \cdot N_\beta \cdot I_\beta$$

(13)

With the assumption of a homogeneous magnetic field in the air gap, the amplitude of the magnetic flux is given by:

$$\Phi_g = B_g \cdot S_g = B_g \cdot h \cdot d$$

(14)

Substituting (2), (13) and (14) in (12) the following equations for calculating the number of turns in each phase are derived:

$$N_a = \frac{3}{\pi} \cdot \sqrt{2} \cdot \frac{B_g \cdot d}{\mu_0 \cdot I_a}$$

(15)

$$N_\beta = \frac{3}{\pi} \cdot \sqrt{2} \cdot \frac{B_g \cdot d}{\mu_0 \cdot I_\beta}$$

(16)

From (15) and (16) it can be seen that required level of magnetic field density in the air gap can be achieved by increasing the number of turns, or by increasing the current.
The required level of magnetic field density is determined from the condition that it only has to spin the egg, and therefore can be much lower than as in conventional electric machines. This level is determined by analyzing a prototype. The results of the experiments show that for rotating magnetic field demonstration, a magnetic flux density of 5-10 mT is large enough to lift and spin a smaller metal egg. Having this range of values in mind, the design problem is to select the appropriate number of turns for each phase winding in order to obtain required magnetic flux density without overheating the coils due to large currents needed to generate this flux density in the large air gap.

If a standard value of copper wire diameter for phase $\alpha$ is selected, with a proper value for current density without overheating, the phase current and the number of turns in phase $\alpha$, using (15) can be calculated. After the number of turns is found, the reactance and resistance of phase $\alpha$ using (3) and (4) and the required transfer ratio using (9) can be calculated. In that case, phase current and number of turns in phase $\beta$ is further calculated as:

$$I_\beta = \frac{I_\alpha}{m}$$

$$N_\beta = m \cdot N_\alpha$$

From (17) the diameter and cross section of copper wire in phase $\beta$ should be adopted to the closest standard values. Since the core has fixed dimensions, it must be must ensured that there is enough space to place both phase windings on the core. The windings will fit on the core if they are wounded in $n$ layers, where $n$ is calculated for each phase independently using:

$$n \geq \frac{2 \cdot N \cdot d_{Cu}}{\pi \cdot d}$$

where the first larger integer value should be adopted. At the end the number of turns for phase $\beta$ is calculated by using the same equations as for phase $\alpha$.

### III. THERMAL DESIGN

Current density of conventional induction motors is typically in the range 3–8 A/mm² [6]. Current density determines the cross section of copper wire, or in other words, the total mass of copper used to wind the inductor. Since the losses depend on the value of current density, the steady state temperature of the windings also depends on current density. In order to prevent overheating, a thermal model is used to select the required value of current density.

#### A. Copper Losses

The design is based on requirement that the steady state temperature of windings does not exceed 80°C. Since the magnetic flux density is in the range of 5-10 mT, the iron losses can be neglected, and only the copper losses are considered.

The total copper losses in both phases at operating temperature $\theta$ are:

$$P_{Cu} = R_\alpha \cdot I_\alpha^2 + R_\beta \cdot I_\beta^2 = 2 \cdot R_\alpha \cdot I_\alpha^2$$

If it is assumed that operating temperature should be $\theta = 80^\circ$C, the resistance should be increased using (7) and the following equation for copper losses in common electrical machines is obtained [6]:

$$P_{Cu} = \frac{\rho_{Cu}}{\gamma_{Cu}} \cdot m_{Cu} \cdot \Delta^2 \approx 2.42 \cdot m_{Cu} \cdot \Delta^2$$

where $\Delta$ is given in A/mm² and total mass of copper is:

$$m_{Cu} = \gamma_{Cu} \cdot V_{Cu} = \gamma_{Cu} \cdot \left( L_{Cu\alpha} \cdot S_{Cu\alpha} + L_{Cu\beta} \cdot S_{Cu\beta} \right)$$

From (21) the losses per cubic meter of copper depend only on the current density:

$$\frac{P_{Cu}}{V_{Cu}} \approx 2.42 \cdot \gamma_{Cu} \cdot \Delta^2 = 21538 \cdot \Delta^2$$

If the copper losses from (21) are combined with (4) and (15) or (16), the following equation is derived:

$$P_{Cu} = \frac{6 \sqrt{2}}{\pi} \cdot \frac{\rho_{Cu} \cdot d \cdot l_{Cu}}{\mu_0} \cdot B_\delta \cdot \Delta = k \cdot B_\delta \cdot \Delta$$

in which $k$ is a constructive constant which depends on characteristics of used materials and core dimensions. From (24) the characteristic product of magnetic flux and current density can be found which will provide that copper losses don’t lead to overheating.

#### B. Current Density

The allowed copper loss can be calculated using Newton’s law of cooling [6,8]:

$$P_{max} = \alpha_c \cdot S_{he} \cdot \Delta \theta$$

where convection and radiation heat transfer surface areas and total heat transfer coefficient are:

$$S_{he} = S_{he} + \pi \cdot h \cdot d = \frac{\pi}{4} \left[ 4 \cdot h \cdot (D + d) + D^2 - d^2 \right]$$

$$\alpha_c = \alpha_c + \frac{S_{he}}{S_{he}}$$

Using (25) the copper losses which will lead to steady state temperature rise of $\theta = 80^\circ$C can be calculated, and after that, using (24) the maximum allowed current density for a given value of magnetic flux density is calculated.
The heat transfer coefficient due to radiation is calculated using Stefan–Boltzmann’s law [8]:

$$\alpha_r = \varepsilon \cdot \sigma \cdot \left( \frac{273 + \theta}{\theta - \theta_a} \right)^4 \left( 273 + \theta_a \right)^4$$  \hspace{1cm} (28)

where the emissivity is taken to be 0.2 for new, and close to 1 for old copper conductors. The heat transfer coefficient due to natural convection is calculated using dimensional thermal analysis, involving Prandtl, Grashoff and Nusselt numbers, and is given by following equation [8]:

$$\alpha_c = Nu \frac{\lambda_f}{h}$$  \hspace{1cm} (29)

In (29) Nusselt number is a function of the product of Grashoff and Prandtl number and is calculated using [8]:

$$Nu = A \cdot (Gr \cdot Pr)^n$$  \hspace{1cm} (30)

where values for coefficients $A$ and $n$ depend on product of Grashoff and Prandtl number and are given in Table I [8].

<table>
<thead>
<tr>
<th>$Gr \cdot Pr$</th>
<th>$A$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$, $10^{-3}$</td>
<td>0.675</td>
<td>0.058</td>
</tr>
<tr>
<td>$10^{-5}$, $10^{-4}$</td>
<td>0.889</td>
<td>0.088</td>
</tr>
<tr>
<td>$10^{-1}$, $10^{-2}$</td>
<td>1.020</td>
<td>0.148</td>
</tr>
<tr>
<td>$10^{-3}$, $10^{-2}$</td>
<td>0.850</td>
<td>0.188</td>
</tr>
<tr>
<td>$10^{-4}$, $10^{-3}$</td>
<td>0.480</td>
<td>0.250</td>
</tr>
<tr>
<td>$10^{-5}$, $10^{-4}$</td>
<td>0.125</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Grashoff number takes into account forces due to differences in density of heated air layers and is calculated using following equation [8]:

$$Gr = \frac{g \cdot \beta \cdot \Delta \theta \cdot h^3}{\nu_f^2}$$ \hspace{1cm} (31)

Prandtl number depends on physical properties of air and can be calculated as [8]:

$$Pr = \frac{\gamma_f \cdot \nu_f \cdot c_f}{\lambda_f}$$ \hspace{1cm} (32)

but is often taken to have a constant value of 0.7.

In (29), (30), (31) and (32) thermal conductivity of air, Prandtl number and viscosity of air are functions of average (film) temperature and are calculated as [8]:

$$\lambda_f = 2.42 \cdot 10^{-2} + 7.2 \cdot 10^{-5} \cdot \theta_f$$ \hspace{1cm} (33)

$$Pr = 0.715 - 2.5 \cdot 10^{-4} \cdot \theta_f$$ \hspace{1cm} (34)

$$\nu_f = 1.32 \cdot 10^{-5} + 9.5 \cdot 10^{-8} \cdot \theta_f$$ \hspace{1cm} (35)

Assuming a value of steady state and ambient temperature and using equations (28)-(35), the total heat transfer coefficient is calculated, and value for allowed copper losses is obtained from (25). The allowed current density is then calculated from (24) and the final result of this calculation is showed on diagram in Fig. 5. The diagram shows allowed current density versus operating temperature, for several values of ambient temperature.

![Figure 5. Current density versus operating temperature](image)

It can be seen from diagram on Fig. 5 that for operating temperature of $\theta = 80^\circ$C and ambient temperatures of $15^\circ$C to $30^\circ$C, allowed current density is in the range of 3-4 A/mm². For most likely ambient temperatures of 20°C to 25°C current density is approximately 3.5 A/mm².
IV. RESULTS OF DESIGN

A. Calculation

For demonstration of the proposed calculation of two-phase Tesla inductor a toroidal iron core from an instrument transformer for voltage 258V and current 25A is selected. The dimensions of the iron core are given in Fig. 6 with the values in Table II.

![Figure 6. Dimensions of the used iron core](image)

**TABLE II. CORE DIMENSIONS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>mm</td>
<td>260</td>
</tr>
<tr>
<td>$d$</td>
<td>mm</td>
<td>180</td>
</tr>
<tr>
<td>$h$</td>
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<td>60</td>
</tr>
<tr>
<td>$a$</td>
<td>mm</td>
<td>40</td>
</tr>
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</table>

For selected standard copper wire with diameter of 1 mm for phase $a$, using equations derived in Section II and Section III, with the selected values of 10 mT for magnetic flux density and 3.5 A/mm$^2$ for current density, all the parameters for constructing a two-phase Tesla inductor are calculated and their values are given in Table III.

**TABLE III. RESULTS OF CALCULATION**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{Cu\alpha}$</td>
<td>mm</td>
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<td>$d_{Cu\beta}$</td>
<td>mm</td>
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<tr>
<td>$S_{Cu\alpha}$</td>
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</tr>
<tr>
<td>$S_{Cu\beta}$</td>
<td>mm$^2$</td>
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</tr>
<tr>
<td>$B_t$</td>
<td>mT</td>
<td>10</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>A/mm$^2$</td>
<td>3.5</td>
</tr>
<tr>
<td>$I_a$</td>
<td>A</td>
<td>2.75</td>
</tr>
<tr>
<td>$I_p$</td>
<td>A</td>
<td>1.84</td>
</tr>
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</tr>
<tr>
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<td>turns</td>
<td>1050</td>
</tr>
<tr>
<td>$m$</td>
<td>/</td>
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</tr>
<tr>
<td>$n_p$</td>
<td>layers</td>
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</tr>
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<td>$R_a$</td>
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</tr>
<tr>
<td>$R_p$</td>
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<td>$X_C$</td>
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</tr>
<tr>
<td>$R_C$</td>
<td>$\Omega$</td>
<td>/</td>
</tr>
<tr>
<td>$C$</td>
<td>$\mu F$</td>
<td>160</td>
</tr>
<tr>
<td>$U_a$</td>
<td>V</td>
<td>21</td>
</tr>
<tr>
<td>$U_p$</td>
<td>V</td>
<td>31</td>
</tr>
<tr>
<td>$U_C$</td>
<td>V</td>
<td>37</td>
</tr>
<tr>
<td>$L_{Cu\alpha}$</td>
<td>m</td>
<td>150</td>
</tr>
<tr>
<td>$L_{Cu\beta}$</td>
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<td>220</td>
</tr>
<tr>
<td>$m_{Cu}$</td>
<td>kg</td>
<td>2</td>
</tr>
<tr>
<td>$V_{Cu\alpha}$</td>
<td>m$^3$</td>
<td>2.28×10$^4$</td>
</tr>
<tr>
<td>$P_{cu}$</td>
<td>W</td>
<td>63.3</td>
</tr>
<tr>
<td>$P_{cu}/V_{cu}$</td>
<td>W/m$^3$</td>
<td>277840</td>
</tr>
</tbody>
</table>

It should be noted that total wire lengths in each phase should be increased for approximately 10% in order to take into account side connections and terminal connections. In this way, the total mass and volume of copper will be slightly increased.
B. Electromagnetic Simulation

For electromagnetic and thermal simulation, FEMM software is used. Fig. 7 shows magnetic flux density plot from phase $\alpha$, at the moment when the current in phase $\alpha$ is at its maximum, while current in phase $\beta$ is equal to zero. This justifies the assumption of homogeneous magnetic filed in the air gap. Fig. 8 shows the similar results for phase $\beta$. From Fig. 7 and Fig. 8 it can be concluded that the magnetic fields are orthogonal, thus creating a rotating magnetic field in the air gap.

![Figure 7. Magnetic flux density plot for phase $\alpha$.](image)

![Figure 8. Magnetic flux density plot for phase $\beta$.](image)

It can be seen that the calculated temperature distribution along air gap is almost equal to the designed temperature of 80°C. During simulation an ambient temperature of 20°C was assumed.

![Figure 9. Magnetic flux density distribution along air gap](image)

![Figure 10. Temperature distribution](image)

C. Thermal Simulation

The electromagnetic FEM simulation considered a planar problem, while the thermal analysis should consider a axisymmetric problem. The copper winding is modeled with a hollow torus, with the same volume as calculated in Table II, while copper losses per cubic meter are calculated from (23). The calculated temperature distribution is showed in Fig. 10 from which it can be seen that the highest temperature is 354 K or 81°C, which is almost equal to the designed temperature of 80°C. During simulation an ambient temperature of 20°C was assumed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
<th>Calculation error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>$\Omega$</td>
<td>4.13</td>
<td>0.50</td>
</tr>
<tr>
<td>$R_f$</td>
<td>$\Omega$</td>
<td>9.61</td>
<td>1.25</td>
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<td>$X_s$</td>
<td>$\Omega$</td>
<td>6.30</td>
<td>2.38</td>
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<tr>
<td>$X_f$</td>
<td>$\Omega$</td>
<td>14.00</td>
<td>2.36</td>
</tr>
</tbody>
</table>
It may seem that the model shown in Fig. 10 is oversimplified, but with current level of design, without experimental verification, it would be unreliable to assume special insulation types for conductors and special wooden or metal platforms for the metal egg to spin on them which may influence the basic calculation.

V. CONCLUSIONS

In this paper a full design procedure for calculating the parameters of Tesla’s two-phase inductor is explained. Starting from basic concepts of electrical machine design, a new method dedicated to special machines with large air gaps and toroidal inductors is developed. The main problem of the proposed design procedure is found to be the selection of magnetic flux density and current density values. While magnetic flux density, required to lift a small metal egg, is determined experimentally, the value for current density is selected as a result of detailed thermal analysis. Even such analysis implies complicated set of equations and is parameter sensitive, it is shown that for reasonable values of operating and ambient temperature, the current density is found to be in a relatively small range of values. It is shown that the analytical model gives very similar results as the FEM computer simulation. It should also be noted that the total copper losses of such a device are around 60 W which is equivalent to a light bulb. Even it seems that the larger values of current densities can be allowed without overheating the winding, the FEM simulation showed that with only 60 W of losses the temperature rise is close to given 80°C limit. The next step would be to build the prototype based on the proposed calculation. Only by experimental verification, the design process will be completed.

REFERENCES